Online Appendix for:

Partial Equilibrium Thinking, Extrapolation, and Bubbles

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B Partially Revealing Prices

When prices are fully revealing, the extrapolation parameter used by PET agents is decreasing in informed agents' informational edge. In this section, we study how the extrapolation parameter changes if we allow for noise, so that prices are no longer fully revealing.

B.0.1 Stochastic Supply and Information Structure

To consider the effect of noise on PET agents' inference problem, we assume that the supply of the risky asset is stochastic, and given by $z_t \overset{iid}{\sim} N(Z, \sigma_z^2)$. To illustrate the effect of noise in the simplest possible way, we assume that agents learn the realization of the supply of the risky asset after two periods. In each period t, all agents are uncertain about $z_{t-j} \overset{iid}{\sim} N(Z, \sigma_z^2)$ for $j \leq 1$ and they know the realization of z_{t-h} for $h \geq 2$. Even though one period lagged prices are partially revealing, this assumption makes prices fully revealing at further lags, thus simplifying PET agents' inference.

B.0.2 Inference Problem with Noise

When prices are fully revealing, uninformed agents think they can extract from prices the exact information that informed agents received in the previous period. This is no longer true when prices are partially revealing, as uninformed agents can only infer a noisy signal of fundamentals from prices. Specifically, in normal times, uninformed agents think that prices take the following form:

$$P_{t-1} = \tilde{a}\left(\tilde{\mathbb{E}}_{I,t-2}[D_T] + \tilde{u}_{t-1}\right) + \tilde{b}\bar{D} - \tilde{c}z_{t-1} \tag{B.1}$$

where $\tilde{a} = \frac{\phi\tilde{\tau}_I}{\phi\tilde{\tau}_I + (1-\phi)\tilde{\tau}_U}$, $\tilde{b} = \frac{(1-\phi)\tilde{\tau}_U}{\phi\tilde{\tau}_I + (1-\phi)\tilde{\tau}_U}$ and $\tilde{c} = \frac{\mathcal{A}}{\phi\tilde{\tau}_I + (1-\phi)\tilde{\tau}_U}$. Since prices are fully revealing in period $t-2$, but they are partially revealing in period $t-1$, uninformed agents extract the following noisy signal from prices:[1]

$$\frac{P_{t-1} - \tilde{a}\tilde{D}_{t-2} - \tilde{b}\bar{D} + \tilde{c}Z}{\tilde{a}} = \tilde{u}_{t-1} - \frac{\tilde{c}}{\tilde{a}}\left(z_{t-1} - Z\right) \tag{B.2}$$

and we can re-write this more simply as:

$$\left(\frac{1}{\tilde{a}}\right)\left(P_{t-1} - \mathbb{E}_{t-1}[P_{t-1}]\right) = \tilde{u}_{t-1} - \frac{\tilde{c}}{\tilde{a}}\left(z_{t-1} - Z\right) \tag{B.3}$$

This shows that uninformed agents are now uncertain as to whether the unexpected price change they observe is due to new information, or to changes in the stochastic supply of the risky asset. Either way, PET agents still extrapolate past prices to recover a (noisy) signal from them.

Given the noisy information that uninformed agents extract from prices, their beliefs in period t are given by:

$$\mathbb{E}_{U,t}[D_T] = \tilde{D}_{t-2} + \left(\frac{\sigma_u^2}{\sigma_u^2 + \left(\frac{\tilde{c}}{\tilde{a}}\right)^2 \sigma_z^2}\right)\left(\frac{1}{\tilde{a}}\right)\left(P_{t-1} - \mathbb{E}_{U,t-1}[P_{t-1}]\right) \tag{B.4}$$

$$= \tilde{D}_{t-2} + \frac{\kappa}{\tilde{a}}\left(P_{t-1} - \mathbb{E}_{U,t-1}[P_{t-1}]\right) \tag{B.5}$$

where $\kappa = \left(\frac{\sigma_u^2}{\sigma_u^2 + \left(\frac{\tilde{c}}{\tilde{a}}\right)^2 \sigma_z^2}\right) \leq 1$ is the weight that PET agents put on the noisy signal they extract from past prices. This shows that the extrapolation parameter θ now depends on two components:

$$\theta \equiv \frac{\kappa}{\tilde{a}} = \underbrace{\left(\frac{\sigma_u^2}{\sigma_u^2 + \left(\frac{1}{\phi\tilde{\tau}_I}\right)^2 \sigma_z^2}\right)}_{\text{weight}}\underbrace{\left(1 + \left(\frac{1-\phi}{\phi}\right)\frac{\tilde{\tau}_U}{\tilde{\tau}_I}\right)}_{\text{inference}} \tag{B.6}$$

where $(\tilde{\tau}_U)^{-1} = \left(\frac{1}{1-\beta^2}\right)\sigma_u^2 = (\tilde{\tau}_I)^{-1} + \sigma_u^2$ and $(\tilde{\tau}_I)^{-1} = \left(\frac{\beta^2}{1-\beta^2}\right)\sigma_u^2$. Starting from the second component in (B.6), $1/\tilde{a}$ is the extrapolation parameter that would prevail if $\sigma_z^2 = 0$ and

[1] The assumption that prices are fully revealing in period $t-2$ means that uninformed agents think they know the exact value of $\tilde{\mathbb{E}}_{I,t-2}[D_T] = \tilde{D}_{t-2}$, as opposed to being uncertain about it.

prices were fully revealing: the more sensitive prices are to shocks, the less strongly do PET agents need to extrapolate unexpected price changes to recover the (in their mind unbiased) noisy signal $\tilde{u}_{t-1} - \frac{\tilde{c}}{\tilde{a}}(z_{t-1} - Z)$ from prices. Turning to the first component in (B.6), $\kappa \leq 1$ is the weight that PET agents put on the information they extract from prices when forming their posterior beliefs. Whenever $\sigma_z^2 > 0$, $\kappa < 1$, and PET agents extrapolate prices less strongly than when prices are fully revealing, and this simply reflects the noisy nature of the signal they are able to infer from prices.

To draw comparative statics, we can substitute the expressions for $\tilde{\tau}_I$ and $\tilde{\tau}_U$ into (B.6), and re-write the extrapolation parameter in terms of the primitives of the model:

$$\theta = \frac{\kappa}{\tilde{a}} = \underbrace{\left(\frac{1}{1 + \left(\frac{1}{\phi}\right)^2 \left(\frac{\beta^2}{1-\beta^2}\right)^2 \sigma_u^2 \sigma_z^2} \right)}_{\text{weight}} \underbrace{\left(1 + \left(\frac{1-\phi}{\phi} \right) \beta^2 \right)}_{\text{inference}} \tag{B.7}$$

From this expression, we see that the extrapolation parameter is decreasing in all sources of noise (σ_u^2 and σ_z^2), as this reduces the informativeness of the signal uninformed agents extract from prices.

On the other hand, increasing the perceived information advantage ($1/\beta^2$) and the fraction of informed agents in the market (ϕ) both have two competing roles. Increasing $1/\beta^2$ (or ϕ) decreases the fully revealing extrapolation parameter $1/\tilde{a}$ as prices are more sensitive to news, but it also increases the weight κ, as prices are a more informative signal. For small enough noise, the first effect dominates, and the extrapolation parameter is decreasing in the informational edge, and in the fraction of informed agents in the market. On the other hand, if there is too much noise in prices, the second effect dominates and the comparative statics are reversed.[2]

[2]Notice that it is a more general property of learning models that the effects of learning are dampened when noise is greater. Therefore, in this section we see that in circumstances where learning is relevant, the comparative statics described in the main text still hold.

C Asymmetric Bubbles and Crashes

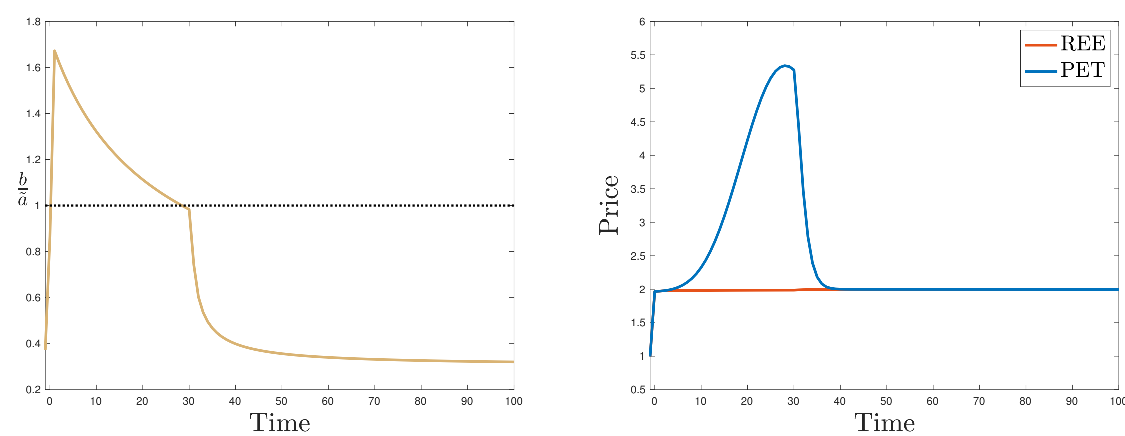
C.1 Slow Boom, and Faster Crash

In the baseline model presented in the core of the paper, we assumed that the precision of each incremental piece of news was constant. Here, we check the robustness of our results if we instead assume that signals are very noisy at first, but become more precise after a certain amount of time. Specifically, we simulate a situation where for the first 30 periods, signals are of precision τ_s, and are of precision $\tau_s' > \tau_s$ afterwards. Figure 1 shows how a bubble and a crash still take place, but the crash is accelerated by the increased precision of signals. Intuitively, this is simply because a high τ_s' makes the feedback effect decrease more rapidly with time.

Figure 1: Asymmetric bubbles and crashes. Starting from a normal times steady state, a displacement $\omega \sim N(\mu_0, \tau_0^{-1})$ is announced in period $t = 0$. Informed agents then receive a signal $s_t = \omega + \epsilon_t$ with $\epsilon \sim N(0, \tau_{s,t}^{-1})$ each period, where $\epsilon_1 > 0$ and $\epsilon_t = 0 \ \forall t > 1$. Moreover, $\tau_{s,t} = \tau_s$ for $t \leq 30$ and $\tau_{s,t} = \tau_s' > \tau_s$ for $t > 30$, which reflects that information is revealed at a faster rate once the bubble bursts. The left panel illustrates the evolution of the strength of the feedback effect. The right panel illustrates the evolution of equilibrium prices, which now exhibit a slower boom and a faster crash.

(a) Strength of the feedback effect

(b) Price



C.2 Misunderstanding the Frequency of Information Arrival

By assuming that informed agents receive new information in each period following a displacement, we are implicitly assuming that uninformed agents understand the frequency with which informed agents receive new information. However, if we change the frequency of information arrival, the true confidence of informed agents becomes decoupled from uninformed agents' perception of it.

In our model, following a displacement, uninformed agents observe a price change in each period, and they attribute each price change to new information. Regardless of the frequency of information arrival, having observed t price changes after t periods, uninformed agents' perception of informed agents' confidence is given by:

$$\tilde{\tau}_{I,t} = \left(\mathbb{V}_{I,0} + (t\tau_s + \tau_0)^{-1}\right)^{-1} \tag{C.8}$$

If informed agents receive news in each period, then $\tilde{\tau}_{I,t} = \tau_{I,t}$. Suppose instead that after t period, informed agents have received only $n_t < t$ signals. Their true confidence is now given by:

$$\tau_{I,t} = \left(\mathbb{V}_{I,0} + (n_t\tau_s + \tau_0)^{-1}\right)^{-1} < \tilde{\tau}_{I,t} \tag{C.9}$$

With this information structure, informed agents need to receive only a finite number of signals for the bubble to burst. Let n_∞ be the total number of signals informed agents receive about the displacement over the whole lifetime of the asset. Long run stability then requires:

$$n_\infty > \bar{n} \tag{C.10}$$

where $\bar{n} = \frac{1}{\tau_s}\left(\frac{1}{\mathbb{V}_{I,0}(\tilde{\zeta}_\infty\zeta_0 - 1)} - \tau_0\right)$, and $\tilde{\zeta}_\infty = \lim_{t\to\infty}\tilde{\zeta}_t$. This implies that bubbles may burst even if the true confidence of informed agents is lower than the true confidence of uninformed agents. This is not the case with models of constant price extrapolation, which instead rely on changes in the true relative confidence of informed and uninformed agents in order to generate bubbles and crashes.

To illustrate this point, Figure 2 shows the response of the economy if informed agents receive a single signal in period $t = 1$, and then receive no further information about the displacement thereafter, so that $n_\infty = 1$. When this is the case, the confidence of uninformed agents rises relative to the confidence of uninformed agents, as shown in the top left panel of Figure 2. However, even though the influence on prices of uninformed agents' biased beliefs rises over time, the economy can still return to a stable region because the strength with which PET agents extrapolate past prices falls over time. Intuitively, PET agents still attribute any price change they observe to additional news about the displacement, and thus think that informed agents' edge is rising over time. Comparing the path of equilibrium prices in the bottom right panel of Figure 2 to the one in Figure 2 we see that when informed agents receive a single shock, the bubble is much more accentuated and takes much longer to die out as the market spends more time in the unstable region. However, the key take-away is that a time-varying extrapolation coefficient allows for bubbles and endogenous crashes that are not driven by changes in agents' relative confidence levels, which would instead be necessary with constant price-extrapolation.

D Alternative Setups

This section presents three alternative setups (temporary shocks, permanent shocks with random walks, and a non-stationary process). We show how the results we uncovered in our main model are robust to altering the assumptions driving the fundamental process. We also illustrate that partial equilibrium thinking leads to momentum and reversals following a temporary shock.

D.1 Temporary Shocks

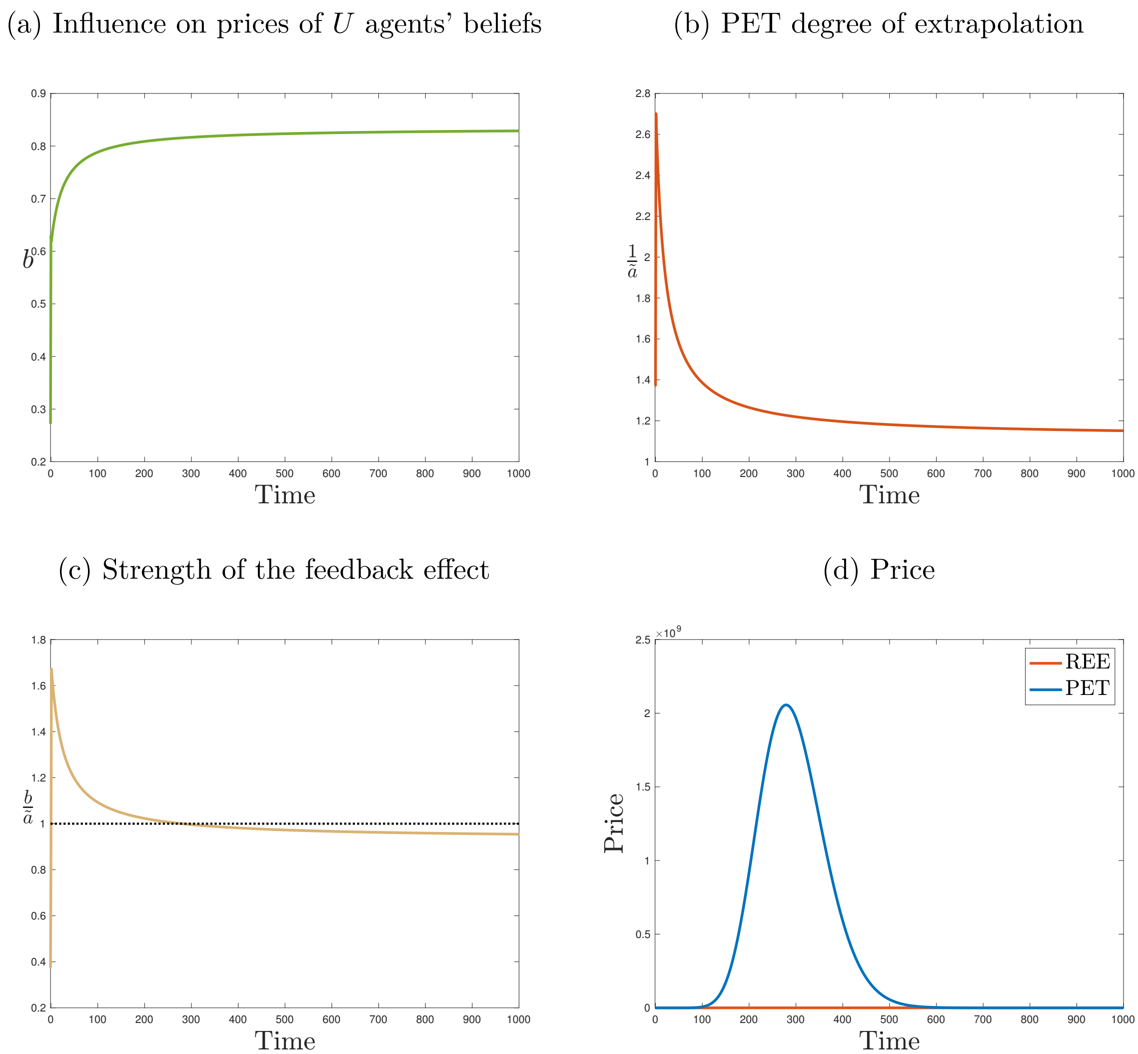
D.1.1 Setup

Assets. We consider an identical economy as our model in the main text, except that the risky asset now pays off a stream of dividends v_t each period, where v follows an AR(1) process:

$$v_t = (1-\rho)\bar{v} + \rho v_{t-1} + u_t \tag{D.1}$$

where $\bar{v}$ is the unconditional mean of the fundamental value of the asset, $\rho \in [0, 1]$ is the persistence coefficient, and $u_t \sim N(0, \tau_u^{-1})$.

Figure 2: Response of the economy when informed agents receive a single signal in period $t = 1$, and no further information thereafter. Starting from a normal times steady state, a displacement $\omega \sim N(\mu_0, \tau_0^{-1})$ is announced in period $t = 0$, and then informed agents receive a *single* signal $s_1 = \omega + \epsilon_1$ with $\epsilon_1 > 0$ and no more signals thereafter. Panels (a) and (b) show how the components of the feedback effect vary over time given this information structure, and Panels (c) and (d) show the evolution of the strength of the feedback effect and of equilibrium prices. Even though b rises over time, the degree of extrapolation still falls after its initial rise, thus allowing the strength of the feedback effect to return to a stable region ($^b/_{\bar{a}} < 1$). Panel (d) shows that the bubble is much more accentuated than the one in Figure 2, as the economy spends longer in the unstable region.



(a) Influence on prices of U agents' beliefs

(b) PET degree of extrapolation

(c) Strength of the feedback effect

(d) Price

Agents and Preferences. There is a continuum of measure one of overlapping agents. All agents live for one period. There are no bequest motives, so agents are myopic. Moreover, we assume that all agents are only concerned with forecasting the fundamental value of the asset, so that at time t they have the following demand function for the risky asset:

$$X_{it} = \frac{\mathbb{E}_{it}[v_{t+1}] - P_t}{AVar_{it}[v_{t+1}]} \tag{D.2}$$

where $\mathbb{E}_{it}[\cdot]$ and $Var_{it}[\cdot]$ characterize agent i's beliefs about next period fundamental payoff given the information set they have at time t.[3] Moreover, we assume that uninformed agents do not observe the history of v_t and they only observe their own realized payoff once they leave the market in period $t+1$.

Information Structure in Normal Times. All agents know $\bar{v}$, as well as all other parameters of the unconditional distribution of v_t and u_t. Moreover, a fraction ϕ of agents are informed, and they observe the whole history u_j for $j \leq t$ before making their portfolio choice in each period. A fraction $(1-\phi)$ of agents are uninformed, and they do not observe u_t, v_t nor their history. However, they can learn information from past prices.

Equilibrium. In equilibrium, uninformed agents' beliefs must be consistent with past prices they observe, given their model of the world. Moreover, all agents trade according to their demand functions in (D.2) given their beliefs, and markets clear. This gives the following price function, conditional on agents' beliefs:

$$P_t = a_t \mathbb{E}_{I,t}[v_{t+1}] + b_t \mathbb{E}_{U,t}[v_{t+1}] - c_t \tag{D.3}$$

where $a_t \equiv \left(\frac{\phi \mathbb{V}_{U,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$, $b_t \equiv \left(\frac{(1-\phi)\mathbb{V}_{I,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$ and $c_t \equiv \left(\frac{\mathbb{V}_{I,t}\mathbb{V}_{U,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right) AZ$ $\mathbb{V}_{i,t} = \mathrm{Var}_{i,t}[v_{t+1}]$ for $i \in \{I, U\}$. Therefore, in order to find the equilibrium price, we need to pin down informed and uninformed agents' beliefs about v_{t+1}.

D.1.2 Normal Times

Informed Agents' Beliefs. Informed agents' beliefs are simply given by:

$$\mathbb{E}_{I,t}[v_{t+1}] = (1-\rho)\bar{v} + \rho v_t \tag{D.4}$$

[3] As in our main framework, this is made such as to keep the analysis as simple as possible. Speculative motives are however studied in Section 3, where we showed that the effects are amplified when informed agents take advantage of uninformed agents' misinference.

$$\mathbb{V}_{I,t}[v_{t+1}] = \sigma_u^2 \tag{D.5}$$

Uninformed Agents' Beliefs. To compute uninformed agents' beliefs, we start by determining what information they extract from past prices.

Misspecified Mapping used to Extract Information from Past Prices. To construct this mapping, we need to write down uninformed agents' beliefs of the price function which generates the prices they observe. This, in turn, requires us to specify uninformed agents' beliefs of other agents' beliefs about next period fundamentals. We denote by $\tilde{}$ uninformed agents' beliefs about a variable. When agents think in partial equilibrium, they think that informed agents hold the following posterior beliefs:

$$\tilde{\mathbb{E}}_{I,t-1}[v_t] = (1-\rho)\bar{v} + \rho\tilde{v}_{t-1} \tag{D.6}$$

$$\tilde{\mathbb{V}}_{I,t-1}[v_t] = \sigma_u^2 \tag{D.7}$$

Moreover, PET agents think that all other uninformed agents do not learn information from prices, and instead trade on the unconditional mean and variance:

$$\tilde{\mathbb{E}}_{U,t-1}[v_t] = \bar{v} \tag{D.8}$$

$$\tilde{\mathbb{V}}_{U,t-1}[v_t] = \frac{\sigma_u^2}{1-\rho^2} \tag{D.9}$$

Substituting these expressions into (D.3), we obtain the price function which uninformed agents think is generating the price that they observe.

$$P_{t-1} = \tilde{a}\left((1-\rho)\bar{v} + \rho\tilde{v}_{t-1}\right) + \tilde{b}\bar{v} - \tilde{c} \tag{D.10}$$

where $\tilde{a} \equiv \frac{\phi\tilde{\mathbb{V}}_{U,t}}{\phi\tilde{\mathbb{V}}_{U,t}+(1-\phi)\tilde{V}_{I,t}} = \frac{\phi\left(\frac{\sigma_u^2}{1-\rho^2}\right)}{\phi\left(\frac{\sigma_u^2}{1-\rho^2}\right)+(1-\phi)\sigma_u^2}$, $\tilde{b} \equiv \frac{(1-\phi)\tilde{\mathbb{V}}_{I,t}}{\phi\tilde{\mathbb{V}}_{U,t}+(1-\phi)\tilde{V}_{I,t}} = \frac{(1-\phi)\sigma_u^2}{\phi\left(\frac{\sigma_u^2}{1-\rho^2}\right)+(1-\phi)\sigma_u^2}$, $\tilde{c} \equiv \frac{\tilde{\mathbb{V}}_{I,t-1}\tilde{\mathbb{V}}_{U,t-1}}{\phi\tilde{\mathbb{V}}_{U,t}+(1-\phi)\tilde{V}_{I,t}}AZ = \frac{\sigma_u^2\left(\frac{\sigma_u^2}{1-\rho^2}\right)}{\phi\left(\frac{\sigma_u^2}{1-\rho^2}\right)+(1-\phi)\sigma_u^2}AZ$. Therefore, uninformed agents invert (D.10) to extract the following information from prices:

$$(1-\rho)\bar{v}+\rho\tilde{v}_{t-1}=\frac{1}{\tilde{a}}P_{t-1}-\frac{\tilde{b}}{\tilde{a}}\bar{v}+\frac{\tilde{c}}{\tilde{a}} \tag{D.11}$$

Uninformed Agents' Beliefs. Having determined what information uninformed agents extract from past prices they observe, we can compute their beliefs:

$$\begin{aligned}\mathbb{E}_{U,t}[v_{t+1}] =& (1-\rho)\bar{v}+\rho\left((1-\rho)\bar{v}+\rho\tilde{v}_{t-1}\right) && \text{(D.12)}\\ =& \left(\frac{\rho}{\tilde{a}}\right)P_{t-1}+\left(1-\rho-\frac{\rho\tilde{b}}{\tilde{a}}\right)\bar{v}+\frac{\rho\tilde{c}}{\tilde{a}} && \text{(D.13)}\end{aligned}$$

$$\mathbb{V}_{U,t}[v_{t+1}]=(1+\rho^2)\sigma_u^2 \tag{D.14}$$

So uninformed agents' beliefs resemble some form of extrapolation:

$$\mathbb{E}_{U,t}[v_{t+1}]=\theta_1P_{t-1}+\theta_2 \tag{D.15}$$

where:

$$\theta_1=\frac{\rho}{\tilde{a}} \tag{D.16}$$

Equilibrium. Substituting agents' beliefs in (D.4), (D.5), (D.13), (D.14) into (D.3), we obtain the path of equilibrium prices:

$$P_t=\left(\frac{b\rho}{\tilde{a}}\right)P_{t-1}+a(1-\rho)(v_t-\bar{v})+\bar{P}\left(1-\frac{b\rho}{\tilde{a}}\right) \tag{D.17}$$

where $a\equiv\frac{\phi\mathbb{V}_{U,t}}{\phi\mathbb{V}_{U,t}+(1-\phi)\mathbb{V}_{I,t}}=\frac{\phi(1+\rho^2)\sigma_u^2}{\phi(1+\rho^2)\sigma_u^2+(1-\phi)\sigma_u^2}$, $b\equiv\frac{(1-\phi)\mathbb{V}_{I,t}}{\phi\mathbb{V}_{U,t}+(1-\phi)\mathbb{V}_{I,t}}=\frac{(1-\phi)\sigma_u^2}{\phi(1+\rho^2)\sigma_u^2+(1-\phi)\sigma_u^2}$, $c\equiv\frac{\mathbb{V}_{U,t}\mathbb{V}_{I,t}}{\phi\mathbb{V}_{U,t}+(1-\phi)\mathbb{V}_{I,t}}AZ=\frac{\sigma_u^2(1+\rho^2)\sigma_u^2}{\phi(1+\rho^2)\sigma_u^2+(1-\phi)\sigma_u^2}$ and $\bar{P}$ is the unconditional mean of prices when agents think in partial equilibrium, and is such that $\bar{P}\left(1-\frac{b\rho}{\tilde{a}}\right)\equiv\left(a+b\left(1-\rho-\frac{\rho\tilde{b}}{\tilde{a}}\right)\right)\bar{v}+b\frac{\rho\tilde{c}}{\tilde{a}}-c$. Let $\mathbb{L}$ denote the lag operator. Then, using the fact that $(v_t-\bar{v})=(1-\rho\mathbb{L})^{-1}u_t$, and rearranging, we can re-write the dynamics of equilibrium prices as follows:

$$\left(P_t - \bar{P}\right) = \frac{a(1-\rho)}{(1-\rho\mathbb{L})\left(1-\frac{b\rho}{\tilde{a}}\mathbb{L}\right)} u_t \tag{D.18}$$

This makes clear that the equilibrium price follows an AR(2) process. Moreover, for this process to be stationary, we need the roots of the characteristic equation to lie outside the unit circle (this is similar to the stability condition of Proposition 6):

$$\rho < 1 \qquad\qquad \frac{b}{\tilde{a}}\rho < 1 \tag{D.19}$$

Rational Expectations Equilibrium Comparison. We can compare the PET impulse response function to the impulse response function which would arise if agents had rational expectations and were able to extract the correct information from past prices.

In this case, informed agents' beliefs are as in (D.4) and (D.13), while uninformed agents' beliefs are as follows:

$$\mathbb{E}_{U,t}[v_{t+1}] = (1-\rho^2)\bar{v} + \rho^2 v_{t-1} \tag{D.20}$$

and with the same conditional variance as in (D.14). Substituting these beliefs into (D.3), we get the following expression for the path of equilibrium prices:

$$P_t = a((1-\rho)\bar{v} + \rho v_t) + b((1-\rho)\bar{v} + \rho v_{t-1}) - c \tag{D.21}$$

$$= a\rho(v_t - \bar{v}) + b\rho(v_{t-1} - \bar{v}) + (a+b)\bar{v} - c \tag{D.22}$$

We can rewrite this as:

$$(P_t - \bar{P}) = \frac{a\rho\left(1-\frac{b}{a}\mathbb{L}\right)}{1-\rho\mathbb{L}} u_t \tag{D.23}$$

Therefore, with rational expectations, the equilibrium price follows an ARMA(1,1). Moreover, stationarity of an ARMA process depends entirely on the autoregressive parameters, and not on the moving average parameters. Specifically, whenever the roots of $(1-\rho z) = 0$ lie outside the unit circle, this system is stationary. In other words, whenever $\rho < 1$, the rational expectations equilibrium is stationary, while this was not enough to guarantee stationarity of the price dynamics when agents think in partial equilibrium.

Simulation. We simulate the REE and PET equilibrium. We start all three cases from a steady state with $v_0 = \bar{v}$, such that uninformed agents' beliefs are consistent with the prices they observe.

Steady State. For the REE equilibrium, uninformed agents' beliefs in steady state are simply equal to $\mathbb{E}_{U,0}[v_1] = \bar{v}$. On the other hand, for PET agents' beliefs to be consistent with the steady state price they observe, it must be that the steady state extracted fundamental $\tilde{v}_{ss}$ satisfies both these expressions:

$$P_0^{PET} = a\bar{v} + b\left((1-\rho)\bar{v} + \rho\left((1-\rho)\bar{v} + \rho\tilde{v}_{ss}\right)\right) \tag{D.24}$$

$$P_0^{CE} = \tilde{a}\left((1-\rho)\bar{v} + \rho\tilde{v}_{ss}\right) + \tilde{b}\bar{v} - \tilde{c} \tag{D.25}$$

so that:

$$(1-\rho)\bar{v} + \rho\tilde{v}_{ss} = \frac{\left(a + b(1-\rho) - \tilde{b}\right)\bar{v} - c + \tilde{c}}{\tilde{a} - b\rho} \tag{D.26}$$

$$\mathbb{E}_{U,0}^{PET}[v_1] = (1-\rho)\bar{v} + \rho\left(\frac{\left(a + b(1-\rho) - \tilde{b}\right)\bar{v} - c + \tilde{c}}{\tilde{a} - b\rho}\right) \tag{D.27}$$

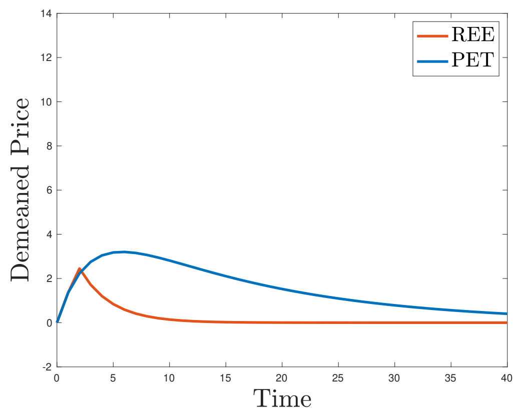
Impulse Response Function. We then shock the economy in period 1 with $u_1 = 5$ and $u_t = 0$ for $t > 1$, and we compute the impulse response function for each equilibrium concept. We plot the demeaned price path to study the response to shocks while taking into account the difference in steady states.

This impulse response function shows PET's ability to generate momentum and reversal to "normal-times" shocks.

D.1.3 Displacement

Information Structure after a Displacement. As before, we model a displacement as an unanticipated and uncertain shock to the unconditional mean of the fundamental value of the asset. Specifically for this setup, we write the evolution of the fundamental

Figure 3: Normal Times Demeaned Price Path. Impulse response function following a shock to the fundamental value of the asset $u_1 = 5$.



value of the asset as follows:

$$\begin{cases} v_t = (1-\rho)\bar{v} + \rho v_{t-1} + u_t & \text{if } t \leq 0 \\ v_t = (1-\rho)(\bar{v}+\omega) + \rho v_{t-1} + u_t & \text{if } t > 0 \end{cases} \tag{D.28}$$

When the displacement is "announced" in period $t = 0$, all agents have the same prior unconditional distribution, $\omega \sim (\mu_0, \tau_0^{-1})$. Starting from period $t = 1$ informed agents receive a signal $s_t = \omega + \epsilon_t$, with $\epsilon_t \sim^{iid} N(0, \tau_s^{-1})$ each period, and they also continue to observe u_t. Uninformed agents do not observe these signals, and still learn information from past prices.

Starting from the steady state equilibrium, let the shock be announced in period $t = 0$, we can then write the evolution of the fundamental value of the asset as follows:

$$v_t = (1-\rho^t)(\bar{v}+\omega) + \rho^t v_0 + \sum_{j=0}^{t-1} \rho^j u_{t-j} \tag{D.29}$$

We can re-write this as:

$$v_t = (1-\rho^t)(\bar{v}+\omega) + \rho^t v_0 + U_{t-1} + u_t \tag{D.30}$$

where $U_{t-1} = \sum_{j=1}^{t-1} \rho^j u_{t-j}$.

Informed Agents' Beliefs. Informed agents' beliefs are given by:

$$\mathbb{E}_{I,t}[v_{t+1}] = (1-\rho^{t+1})\left(\bar{v} + \underbrace{\left(\frac{t\tau_s}{t\tau_s+\tau_0}S_t + \frac{\tau_0}{t\tau_s+\tau_0}\mu_0\right)}_{\mathbb{E}_{I,t}[\omega]}\right) + \rho^{t+1}v_0 + U_t \tag{D.31}$$

$$\mathbb{V}_{I,t}[v_{t+1}] = (1-\rho^{t+1})^2 \underbrace{(t\tau_s+\tau_0)^{-1}}_{\mathbb{V}_{I,t}[\omega]} + \sigma_u^2 \tag{D.32}$$

where $S_t \equiv \sum_{j=1}^{t} s_j$, and since $s_j = \omega + \epsilon_j$, we can re-write this as a stationary AR(1) process with mean ω and AR(1) coefficient $\left(\frac{t-1}{t}\right)$: $(S_t - \omega) = \frac{1}{t\left(1-\left(\frac{t-1}{t}\right)\mathbb{L}\right)}\epsilon_t$.

Uninformed Agents' Beliefs. Turning to uninformed agent's beliefs, we proceed in the same two steps as when solving the model in normal times: first, we determine what unbiased signal uninformed agents extract from prices; second, we determine how they use this information to compute their forecasts about next period fundamentals.

Misspecified Mapping used to Extract Information from Past Prices. Unlike in normal times, uninformed agents now have to gain information about two shocks (u_t and ϵ_t) from prices, and both these shocks are incorporated into prices via informed agents' beliefs. Therefore, uninformed agents extract $\mathbb{E}_{i,t-1}[v_t]$ from P_{t-1}. To do so, they must form beliefs about what generates the prices they observe, which in turn requires them to from beliefs about all other agents' beliefs. Specifically, they correctly understand how informed agents form their beliefs:

$$\tilde{\mathbb{E}}_{I,t-1}[v_t] = (1-\rho^t)\left(\frac{(t-1)\tau_s}{(t-1)\tau_s+\tau_0}S_t + \frac{\tau_0}{(t-1)\tau_s+\tau_0}\mu_0\right) + \rho^t v_0 + U_t \tag{D.33}$$

$$\tilde{\mathbb{V}}_{I,t-1}[v_t] = (1-\rho^t)^2((t-1)\tau_s+\tau_0)^{-1} + \sigma_u^2 \tag{D.34}$$

but they mistakenly think that all other uninformed agents do not infer information from prices:

$$\tilde{\mathbb{E}}_{U,t-1}[v_t] = (1-\rho^t)(\bar{v}+\mu_0) + \rho^t\bar{v} \tag{D.35}$$

$$\tilde{\mathbb{V}}_{U,t-1}[v_t] = (1-\rho^t)^2\tau_0^{-1} + \frac{\sigma_u^2}{1-\rho^2} \tag{D.36}$$

Given these beliefs, they think that market clearing prices are generated by:

$$P_{t-1} = \tilde{a}_{t-1}\tilde{\mathbb{E}}_{I,t-1}[v_t] + \tilde{b}_{t-1}\tilde{\mathbb{E}}_{U,t-1}[v_t] - \tilde{c}_t \tag{D.37}$$

where $\tilde{a}_{t-1} \equiv \frac{\phi\tilde{V}_{U,t-1}}{\phi\tilde{\mathbb{V}}_{U,t-1}+(1-\phi)\tilde{V}_{I,t-1}}$, $\tilde{b}_{t-1} \equiv \frac{(1-\phi)\tilde{V}_{I,t-1}}{\phi\tilde{\mathbb{V}}_{U,t-1}+(1-\phi)\tilde{V}_{I,t-1}}$, $\tilde{c}_{t-1} \equiv \frac{\tilde{V}_{U,t-1}\tilde{V}_{I,t-1}AZ}{\phi\tilde{\mathbb{V}}_{U,t-1}+(1-\phi)\tilde{V}_{I,t-1}}$, and where $\tilde{\mathbb{V}}_{I,t-1}[v_{t+1}]$ and $\tilde{\mathbb{V}}_{U,t-1}[v_{t+1}]$ are given by (D.34) and (D.36) respectively.

Importantly, notice that the mapping that uninformed agents use to extract information from prices is now time-varying (since $\tilde{a}_{t-1}$, $\tilde{b}_{t-1}$ and $\tilde{c}_{t-1}$ are all time-varying). The time variation in these coefficients stems from the fact that uninformed agents understand that displacements generate changes in uncertainty.

Uninformed agents then invert this mapping to infer information from prices:

$$\tilde{\mathbb{E}}_{I,t-1}[v_t] = \frac{1}{\tilde{a}_{t-1}}P_{t-1} - \frac{\tilde{b}_{t-1}}{\tilde{a}_{t-1}}\tilde{\mathbb{E}}_{U,t-1}[v_t] + \frac{\tilde{c}_{t-1}}{\tilde{a}_{t-1}} \tag{D.38}$$

Uninformed Agents' Beliefs. We are now left to pin down how uninformed agents update their beliefs given the information they extract from prices. For ease of notation, let $\tilde{v}_{t|t-1} \equiv \tilde{\mathbb{E}}_{I,t-1}[v_t]$ from (D.38). We can then write this as:

$$\tilde{v}_{t|t-1} = (1-\rho^t)\left(\bar{v} + \left(\frac{(t-1)\tau_s}{(t-1)\tau_s+\tau_0}\left(\omega + \frac{\sum_{j=1}^{t-1}\epsilon_j}{t-1}\right) + \frac{\tau_0}{(t-1)\tau_s+\tau_0}\mu_0\right)\right) + \rho^t v_0 + U_{t-1} \tag{D.39}$$

Uninformed agents' forecasts are then given by:

$$\mathbb{E}_{U,t}[v_{t+1}] = (1-\rho^{t+1})\left(\bar{v} + \mathbb{E}_{U,t}[\omega|\tilde{v}_{t|t-1}]\right) + \rho^{t+1}\tilde{v}_0 + \rho\mathbb{E}_{U,t}[U_{t-1}|\tilde{v}_{t|t-1}] \tag{D.40}$$

$$\begin{aligned}\mathbb{V}_{U,t}[v_{t+1}] = &(1-\rho^{t+1})^2\mathbb{V}_{U,t}[\omega|\tilde{v}_{t|t-1}] + \rho^2\mathbb{V}_{U,t}[U_{t-1}|\tilde{v}_{t|t-1}] \\ &+ 2(1-\rho^{t+1})\rho\mathrm{Cov}_{U,t}[\omega, U_{t-1}|\tilde{v}_{t|t-1}] + (1+\rho^2)\sigma_u^2\end{aligned} \tag{D.41}$$

where:

$$\mathbb{E}_{U,t}\begin{bmatrix}\omega \\ U_{t-1}\end{bmatrix} = \begin{bmatrix}\mathbb{E}[\omega] + \frac{\text{Cov}(\omega,\tilde{v}_{t|t-1})}{\text{Var}(\tilde{v}_{t|t-1})}\left(\tilde{v}_{t|t-1} - \mathbb{E}[\tilde{v}_{t|t-1}]\right) \\ \mathbb{E}[U_{t-1}] + \frac{\text{Cov}(U_{t-1},\tilde{v}_{t|t-1})}{\text{Var}(\tilde{v}_{t|t-1})}\left(\tilde{v}_{t|t-1} - \mathbb{E}[\tilde{v}_{t|t-1}]\right)\end{bmatrix} \tag{D.42}$$

$$\text{Cov}_{U,t}\begin{bmatrix}\omega \\ U_{t-1}\end{bmatrix} = \begin{bmatrix}\text{Var}(\omega) - \frac{\left(\text{Cov}(\omega,\tilde{v}_{t|t-1})\right)^2}{\text{Var}(\tilde{v}_{t|t-1})} & \text{Cov}(w,U_{t-1}) - \frac{\text{Cov}(\omega,\tilde{v}_{t|t-1})\text{Cov}(U_{t-1},\tilde{v}_{t|t-1})}{\text{Var}(\tilde{v}_{t|t-1})} \\ \text{Cov}(w,U_{t-1}) - \frac{\text{Cov}(\omega,\tilde{v}_{t|t-1})\text{Cov}(U_{t-1},\tilde{v}_{t|t-1})}{\text{Var}(\tilde{v}_{t|t-1})} & \text{V}(U_{t-1}) - \frac{\left(\text{Cov}(U_{t-1},\tilde{v}_{t|t-1})\right)^2}{\text{Var}(\tilde{v}_{t|t-1})}\end{bmatrix} \tag{D.43}$$

and

$$\mathbb{E}\begin{bmatrix}\omega \\ U_{t-1} \\ \tilde{v}_{t|t-1}\end{bmatrix} = \begin{bmatrix}\mu_0 \\ 0 \\ (1-\rho^t)\mu_0 + \rho^t\tilde{v}_0\end{bmatrix} \tag{D.44}$$

$$\text{Cov}\begin{bmatrix}\omega \\ U_{t-1} \\ \tilde{v}_{t|t-1}\end{bmatrix} = \begin{bmatrix}\tau_0^{-1} & 0 & (1-\rho^t)\left(\frac{(t-1)\tau_s}{(t-1)\tau_s+\tau_0}\right)\tau_0^{-1} \\ 0 & \left(\frac{1-\rho^{2(t-1)}}{1-\rho^2}\rho^2\sigma_u^2\right) & \left(\frac{1-\rho^{2(t-1)}}{1-\rho^2}\rho^2\sigma_u^2\right) \\ (1-\rho^t)\left(\frac{(t-1)\tau_s}{(t-1)\tau_s+\tau_0}\right)\tau_0^{-1} & \left(\frac{1-\rho^{2(t-1)}}{1-\rho^2}\rho^2\sigma_u^2\right) & (1-\rho^t)^2\left(\frac{(t-1)\tau_s}{(t-1)\tau_s+\tau_0}\right)^2\left(\tau_0^{-1}+((t-1)\tau_s)^{-1}\right)+\left(\frac{1-\rho^{2(t-1)}}{1-\rho^2}\right)\rho^2\sigma_u^2\end{bmatrix} \tag{D.45}$$

Therefore, we can write uninformed agents' beliefs as:

$$\mathbb{E}_{U,t}[v_{t+1}] = \theta_{1,t}P_{t-1} + \theta_{2,t} \tag{D.46}$$

where:

$$\theta_{1,t} = \left((1-\rho^{t+1})\frac{\text{Cov}(\omega,\tilde{v}_{t|t-1})}{\text{Var}(\tilde{v}_{t|t-1})} + \rho\frac{\text{Cov}(\omega,U_{t-1})}{\text{Var}(\tilde{v}_{t|t-1})}\right)\frac{1}{\tilde{a}_{t-1}} \tag{D.47}$$

Equilibrium. Given agents' beliefs, equilibrium prices are given by:

$$\begin{aligned}P_t = C_t + \left((1-\rho^{t+1})\frac{\text{Cov}(\omega,\tilde{v}_{t|t-1})}{\text{Var}(\tilde{v}_{t|t-1})} + \rho\frac{\text{Cov}(\omega,U_{t-1})}{\text{Var}(\tilde{v}_{t|t-1})}\right)\frac{b_t}{\tilde{a}_{t-1}}P_{t-1} \\ + a_t\left(\frac{t\tau_s(1-\rho^{t+1})}{t\tau_s+\tau_0}\right)\frac{1}{t\left(1-\left(\frac{t-1}{t}\right)\mathbb{L}\right)}\epsilon_t\end{aligned} \tag{D.48}$$

$$P_t = C_t + b_t\theta_{1,t}P_{t-1} + a_t\left(\frac{t\tau_s(1-\rho^{t+1})}{t\tau_s+\tau_0}\right)\frac{1}{t\left(1-\left(\frac{t-1}{t}\right)\mathbb{L}\right)}\epsilon_t \tag{D.49}$$

where C_t is deterministic. This resembles an AR(2) process, but this time with time-varying roots.

Rational Expectations Equilibrium Comparison. To solve for the rational expectations equilibrium, we compute similar steps as above, with the one difference that uninformed agents are able to recover $v_{t|t-1} = \mathbb{E}_{1,t-1}[v_t]$ from past prices.

Solving for the equilibrium price, we find that:

$$P_t^{REE} = C_t^{REE} + \left((1-\rho^{t+1})\frac{\text{Cov}(\omega, \tilde{v}_{t|t-1})}{\text{Var}(\tilde{v}_{t|t-1})} + \rho\frac{\text{Cov}(\omega, U_{t-1})}{\text{Var}(\tilde{v}_{t|t-1})}\right)b_t v_{t|t-1} + a_t v_{t+1|t-1} \tag{D.50}$$

$$P_t^{REE} = C_t^{REE} + a_t\left(1-\left((1-\rho^{t+1})\frac{\text{Cov}(\omega, \tilde{v}_{t|t-1})}{\text{Var}(\tilde{v}_{t|t-1})} + \rho\frac{\text{Cov}(\omega, U_{t-1})}{\text{Var}(\tilde{v}_{t|t-1})}\right)\frac{b_t}{a_t}\mathbb{L}\right)v_{t+1|t} \tag{D.51}$$

$$\left(P_t^{REE} - \bar{P}\right) = \left(\frac{a_t(1-\rho^{t+1})t\tau_s}{t\tau_s+\tau_0}\right)\frac{\left(1-\left((1-\rho^{t+1})\frac{\text{Cov}(\omega,\tilde{v}_{t|t-1})}{\text{Var}(\tilde{v}_{t|t-1})} + \rho\frac{\text{Cov}(\omega,U_{t-1})}{\text{Var}(\tilde{v}_{t|t-1})}\right)\frac{b_t}{a_t}\mathbb{L}\right)}{t\left(1-\left(\frac{t-1}{t}\right)\mathbb{L}\right)}\epsilon_t \tag{D.52}$$

so that the REE equilibrium price resembles an ARMA(1,1) process with time-varying coefficients. Once again, notice that the AR roots are always less than one.

Impulse Response Function. We initiate the economy at the same steady state as in normal times. In period $t = 0$, a displacement is announced, and all agents share the same unconditional distribution of the shock to the unconditional mean of the fundamental value of the asset: $\omega \sim N(\mu_0, \tau_0^{-1})$. Finally, starting in period $t = 1$, informed agents receive a signal s_t which is informative about the fundamental value of the asset.

Period $t = 0$. In period $t = 0$ agents learn that starting next period the unconditional mean of the fundamental value of the asset is $\bar{v} + \omega$, where $\omega \sim N(\mu_0, \tau_0^{-1})$. For all equilibrium concepts, informed agents' posterior beliefs are given by:

$$\mathbb{E}_{I,0}[v_1] = (1-\rho)\left(\bar{v}+\mu_0\right) + \rho v_0 \tag{D.53}$$

$$\mathbb{V}_{I,0}[v_1] = (1-\rho)^2(\tau_0)^{-1} + \sigma_u^2 \tag{D.54}$$

Uninformed agents' posterior beliefs differ in the REE and PET equilibrium:

$$\mathbb{E}_{U,0}[v_1] = (1-\rho)\left(\bar{v}+\mu_0\right) + \rho\left((1-\rho)\bar{v} + \rho\tilde{v}_{ss0}\right) \tag{D.55}$$

$$\mathbb{E}^{REE}_{U,0}[v_1] = (1-\rho)\left(\bar{v}+\mu_0\right) + \rho\bar{v} \tag{D.56}$$

$$\mathbb{V}_{U,0}[v_1] = \mathbb{V}^{REE}_{U,0}[v_1] = (1-\rho)^2(\tau_0)^{-1} + (1+\rho^2)\sigma_u^2 \tag{D.57}$$

where $\tilde{v}_{ss0}$ is the same steady state as in the normal times case, in (D.26). Given these beliefs, we can construct P_0 and P_0^{REE} using (D.3), and we can also obtain the mapping that uninformed agents use to extract information from P_0.

Period $t = 1$. Informed agents obtain s_1 and their posterior beliefs are given by:

$$\mathbb{E}_{I,1}[v_2] = (1-\rho^2)\left(\bar{v} + \frac{\tau_s}{\tau_s+\tau_0}S_1 + \frac{\tau_0}{\tau_s+\tau_0}\mu_0\right) + \rho^2 v_0 + \rho u_1 \tag{D.58}$$

$$\mathbb{V}_{I,t}[v_2] = (1-\rho^2)^2(\tau_s+\tau_0)^{-1} + \sigma_u^2 \tag{D.59}$$

Uninformed PET agents learn information about u_0 from P_0 by extracting $\tilde{v}_0$ from prices.

$$\mathbb{E}_{U,1}[v_2] = (1-\rho^2)(\bar{v}+\mu_0) + \rho^2\tilde{v}_0 \tag{D.60}$$

$$\mathbb{V}_{U,1}[v_2] = (1-\rho^2)^2(\tau_0)^{-1} + (1+\rho^2)\sigma_u^2 \tag{D.61}$$

where:

$$\tilde{v}_0 = \frac{P_0 - \tilde{b}_0\mathbb{E}_{0,U}[v_1] + \tilde{c}_0}{\tilde{a}_0} \tag{D.62}$$

Similarly, uninformed agents' beliefs for the REE equilibrium are given by:

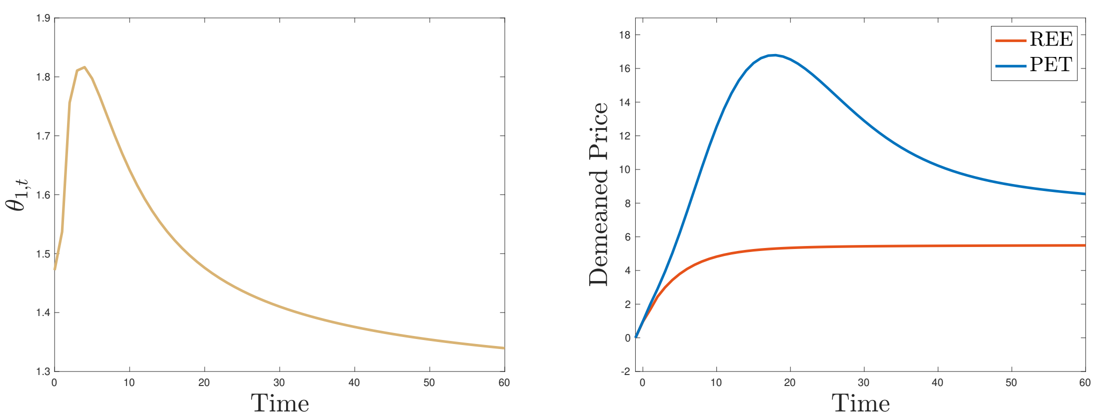
$$\mathbb{E}_{U,1}^{REE}[v_2] = (1-\rho^2)(\bar{v}+\mu_0) + \rho^2 v_0 \tag{D.63}$$

$$\mathbb{V}_{U,1}^{REE}[v_2] = (1-\rho^2)^2(\tau_0)^{-1} + (1+\rho^2)\sigma_u^2 \tag{D.64}$$

Given these beliefs, we can solve for the PET and REE equilibrium prices in period $t = 1$.

Period $t > 1$. Starting in period $t = 2$, uninformed agents gain information about both U_t and S_t by learning from past prices, and the economy evolves as described above.

Figure 4: Displacement Demeaned Price Path and Extrapolation Parameter. Impulse response function following a displacement, modeled as an uncertain shock to the unconditional mean of the process.



D.2 Permanent Shocks - Random Walk Fundamentals

The way we have modeled normal times shocks and displacements in Section D.1 draws a distinction between displacements being permanent shock and normal times shocks as being transitory. In what follows, we instead consider the case where fundamentals evolve according to a random walk, so that both normal times and displacement shocks are permanent.

In both cases, displacement shocks differ to normal times shocks because displacements are shocks for which informed agents gain more information about over time (while normal time shocks are effectively revealed next period, so there is no sense in which agents gradually gain more information about these shocks over time, other than by observing their realization).

D.2.1 Setup

Assets. Consider an identical economy as our model in the main text, except that the fundamental value of the asset evolves according to a random walk:

$$v_t = v_{t-1} + u_t \tag{D.65}$$

where $u_t \sim N(0, \tau_u^{-1})$.

Agents and Preferences. There is a continuum of measure one of agents, and we assume that they are only concerned with forecasting the fundamental value of the asset, so that at time t they have the following demand function for the risky asset:

$$X_{it} = \frac{\mathbb{E}_{it}[v_{t+1}] - P_t}{AVar_{it}[v_{t+1}]} \tag{D.66}$$

where $\mathbb{E}_{it}[\cdot]$ and $Var_{it}[\cdot]$ characterize agent i's beliefs about next period fundamental given the information set they have at time t.

Information Structure in Normal Times. All agents know the unconditional distribution of u_t. Moreover, a fraction ϕ of agents are informed, and they observe the whole history u_j for $j \leq t$ before making their portfolio choice in each period. A fraction $(1-\phi)$ of agents are uninformed, and they do not observe u_t, v_t, nor their history. However, they can infer information from past prices.

Equilibrium. In equilibrium, uninformed agents' beliefs must be consistent with the past prices they observe, given their model of the world. Moreover, all agents trade according to their demand functions in (D.66) given their beliefs, and markets clear. The market clearing price function, conditional on agents' beliefs, is then given by:

$$P_t = a_t \mathbb{E}_{I,t}[v_{t+1}] + b_t \mathbb{E}_{U,t}[v_{t+1}] - c_t \tag{D.67}$$

where $a_t = \left(\frac{\phi \mathbb{V}_{U,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$, $b_t = \left(\frac{(1-\phi)\mathbb{V}_{I,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$, $c_t = \left(\frac{\mathbb{V}_{I,t}\mathbb{V}_{U,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right) AZ$, and $\mathbb{V}_{i,t} = \text{Var}_{i,t}[v_{t+1}]$ for $i \in \{I, U\}$. Therefore, in order to find the equilibrium price, we need to pin down informed and uninformed agents' beliefs about v_{t+1}.

D.2.2 Normal Times

Agents' Beliefs. Informed agents' beliefs are simply given by:

$$\mathbb{E}_{I,t}[v_{t+1}] = v_{t-1} + u_t \tag{D.68}$$

$$\mathbb{V}_I[v_{t+1}] = \sigma_u^2 \tag{D.69}$$

Since prices in our economy are fully revealing, uninformed agents' beliefs are given by:

$$\mathbb{E}_{U,t}[v_{t+1}] = \tilde{v}_{t-1} \tag{D.70}$$

$$\mathbb{V}_U[v_{t+1}] = 2\sigma_u^2 \tag{D.71}$$

where $\tilde{v}_{t-1}$ is uninformed agents' beliefs of previous period fundamental, which they extract from past prices.

Moreover, PET agents think that all other uninformed agents do not learn information from prices, and instead trade on the unconditional mean:

$$\tilde{\mathbb{E}}_{U,t-1}[v_t] = \bar{v} \tag{D.72}$$

We further assume that PET agents believe that other uninformed are trading on some finite variance level:[4]

$$\tilde{\mathbb{V}}_{U,t-1}[v_t] = \tilde{\sigma}_u^2 \geq \sigma_u^2 \tag{D.73}$$

[4] Otherwise, the unconditional variance level is infinite and uninformed agents would not trade.

Substituting these expressions into (D.3), we obtain the price function which uninformed agents think is generating the price that they observe.

$$P_{t-1} = \tilde{a}\tilde{v}_{t-1} + \tilde{b}\bar{v} - \tilde{c} \tag{D.74}$$

where $\tilde{a} \equiv \frac{\phi\tilde{\mathbb{V}}_{U,t}}{\phi\tilde{\mathbb{V}}_{U,t}+(1-\phi)\tilde{\mathbb{V}}_{I,t}} = \frac{\phi\sigma_u^2}{\phi\sigma_u^2+(1-\phi)\tilde{\sigma}_u^2}$, $\tilde{b} \equiv \frac{(1-\phi)\tilde{\mathbb{V}}_{I,t}}{\phi\tilde{\mathbb{V}}_{U,t}+(1-\phi)\tilde{\mathbb{V}}_{I,t}} = \frac{(1-\phi)\tilde{\sigma}_u^2}{\phi\sigma_u^2+(1-\phi)\tilde{\sigma}_u^2}$, $\tilde{c} \equiv \frac{\tilde{\mathbb{V}}_{I,t-1}\tilde{\mathbb{V}}_{U,t-1}}{\phi\tilde{\mathbb{V}}_{U,t}+(1-\phi)\tilde{\mathbb{V}}_{I,t}}AZ = \frac{\tilde{\sigma}_u^2\sigma_u^2}{\phi\sigma_u^2+(1-\phi)\tilde{\sigma}_u^2}AZ$. Therefore, uninformed agents invert (D.74) to extract the following information from prices:

$$\tilde{v}_{t-1} = \frac{1}{\tilde{a}}P_{t-1} - \frac{\tilde{b}}{\tilde{a}}\bar{v} + \frac{\tilde{c}}{\tilde{a}} \tag{D.75}$$

We can then simply define the price extrapolation coefficient as $\theta = \frac{1}{\tilde{a}}$, and the risk-premium coefficient:

$$\delta = \frac{\tilde{b}}{\tilde{a}}\bar{v} - \frac{\tilde{c}}{\tilde{a}} \tag{D.76}$$

such that:

$$\tilde{v}_{t-1} = \theta P_{t-1} - \delta \tag{D.77}$$

Equilibrium. By substituting these expressions for agents' beliefs in (D.67), we find that equilibrium prices evolve according to:

$$P_t = av_t + b\theta P_{t-1} + b\theta\delta - c \tag{D.78}$$

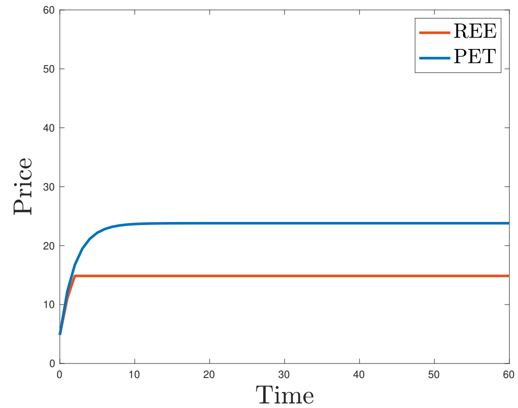
Starting from a steady state where the fundamental value of the asset is constant at v_0, if we study the impulse response function to a shock $u_1 \neq 0$, we have that:

$$P_t = \sum_{j=1}^{t-1}(\beta\theta)^j(av_1 + b\theta\delta - c) + (\beta\theta)^t \tag{D.79}$$

The economy will converge to a new steady state if and only if $\beta\theta < 1$. Otherwise, prices and uninformed agents' beliefs become extreme and decoupled from fundamentals.

Impulse Response Function. We plot the impulse response function in Figure 5. Following a normal times shock, PET leads to momentum as delayed over-reaction.

Figure 5: Path of equilibrium prices in normal times.



D.2.3 Displacement

Displacement Shock and Information Structure. We model a displacement as a one-off shock to fundamentals, ω, whose realization no agent can observe. Instead, agents have a prior distribution of $\omega \sim N(\mu_0, \tau_0^{-1})$. The shock is announced in period $t = 0$, and comes into effect in period $t = 1$.

$$v_t = v_0 + \omega + \sum_{j=1}^{t} u_t \tag{D.80}$$

Starting in period $t = 1$, all informed agents receive a common signal $s_t = \omega + \epsilon_t$ where $\epsilon_t \sim N(0.\tau_s^{-1})$. Uninformed agents do not see these signals, but can still learn information from past prices.

Agents' Beliefs. In period $t = 0$, when the displacement is announced, agents' beliefs are as follows:

$$\mathbb{E}_{I,0}[v_1] = v_{-1} + \mu_0 + u_0 \tag{D.81}$$

$$\mathbb{V}_{I,0}[v_1] = \tau_0^{-1} + \sigma_u^2 \tag{D.82}$$

$$\mathbb{E}_{U,0}[v_1] = \tilde{v}_{-1} + \mu_0 \tag{D.83}$$

$$\mathbb{V}_{U,0}[v_1] = \tau_0^{-1} + 2\sigma_u^2 \tag{D.84}$$

Starting in period $t = 1$, agents' beliefs are given by:

$$\mathbb{E}_{I,t}[v_{t+1}] = v_0 + \left(\frac{t\tau_s}{t\tau_s + \tau_0} S_t + \frac{\tau_0}{t\tau_s + \tau_0}\mu_0\right) + \sum_{j=1}^{t} u_j \tag{D.85}$$

$$\mathbb{V}_{I,t}[v_{t+1}] = (t\tau_s + \tau_0)^{-1} + \sigma_u^2 \tag{D.86}$$

$$\mathbb{E}_{U,t}[v_{t+1}] = \tilde{\mathbb{E}}_{I,t-1}[v_t] \tag{D.87}$$

$$\mathbb{V}_{U,t}[v_{t+1}] = \left(\mathbb{V}_{U,t-1}[v_t] + \sigma_u^2\right) - \frac{\left(\left(\frac{(t-1)\tau_s}{(t-1)\tau_s+\tau_0}\right)\tau_0^{-1} + (t-1)\sigma_u^2\right)^2}{\left(\frac{(t-1)\tau_s}{(t-1)\tau_s+\tau_0}\right)^2\left(\tau_0^{-1} + ((t-1)\tau_s)^{-1}\right) + (t-1)\sigma_u^2} \tag{D.88}$$

Once again, we need to specify what information uninformed agents extract from prices. When agents think in partial equilibrium, we can write their beliefs as follows:

$$\mathbb{E}_{U,t}[v_{t+1}] = \theta_t P_{t-1} + \theta_t \delta_t \tag{D.89}$$

where PET provides a micro-foundation for the time-varying extrapolation coefficients.

We solve for PET using the same steps as in the rest of the paper. The one step that requires us to make additional assumptions regards the uncertainty faced by uninformed agents. Specifically, the unconditional variance of the process for fundamentals is infinity given the process is a random walk. Instead, we assume that cursed agents have the same variance as uninformed PET agents.

Equilibrium. If we turn off all normal time shocks, on average, in equilibrium, prices evolve as follows:

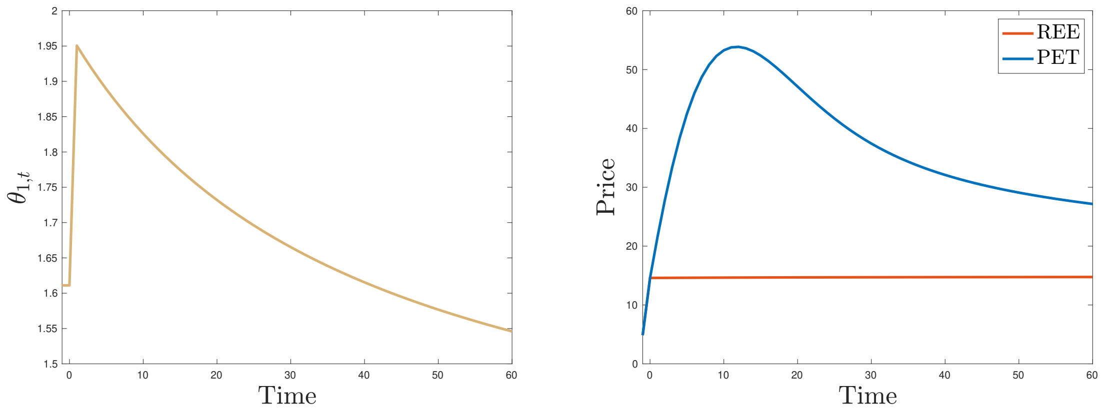
$$P_t = a_t(v_0 + \omega) + b_t\theta_t P_{t-1} + b_t\theta_t\delta_t - c_t \tag{D.90}$$

For simplicity, let $\delta_t = c_t = v_0 = 0$. Then, we can write prices as:

$$P_t = \left(a_t + \sum_{j=1}^{t-1}\prod_{i=1}^{j}\left(\theta_t b_{t+1-i}\right) a_{t-j}\right)\omega + \prod_{j=1}^{t}\left(\theta_t b_{t+1-j}\right) P_0 \tag{D.91}$$

Impulse Response Function. We plot the impulse response function of the displacement shock in Figure 6. Following a displacement, the degree of extrapolation is initially stronger, and then declines over time, leading to bubbles and endogenous crashes.

Figure 6: Time-variation in the extrapolation parameter (left panel) and path of equilibrium prices (right panel) following a displacement.



D.3 Unobservable Growth Rate of Dividends

This section considers an alternative setup, where uninformed agents can observe dividends, and they are instead learning about the unobservable growth rate of dividends.

Fundamentals and Shocks. The setup is similar as previously and in the main text, except that the asset pays dividends each periods, and the dividend process follows:

$$D_{t+1} = D_t + g_{t+1} + \xi_{t+1} \tag{D.92}$$

where the growth rate is evolving according to:

$$g_{t+1} = (1-\rho)\bar{g} + \rho g_t + u_{t+1} \tag{D.93}$$

where $\xi_{t+1} \sim N(0, \sigma_\xi^2)$ and $u_{t+1} \sim N(0, \sigma_u^2)$. Following a displacement, the process for dividend growth is shocked such that:

$$g_{t+1} = (1-\rho)\bar{g} + \rho g_t + \omega + u_{t+1} \tag{D.94}$$

where $\omega \sim N(\mu_0, \tau_0^{-1})$. Therefore this displacement shock is equivalent to shocking the unconditional mean of the growth rate of dividends by $\left(\frac{\omega}{1-\rho}\right)$.

Agents and Preferences. We consider an OLG economy where all agents live for one period, and have the following demand function for the risky asset:

$$X_{it} = \frac{\mathbb{E}_{it}[D_{t+1}] - P_t}{A\mathbb{V}_{it}[D_{t+1}]} \tag{D.95}$$

In this economy, agents are concerned with next period payoff, but we shut down speculative motives to keep things tractable.

Information Structure. In normal times, all agents know $\bar{g}$, ρ, the distribution of ξ_t and u_t, and all agents also observe D_t. Moreover, informed agents observe u_{t+1}. Uninformed agents can learn information from past prices.

Displacements are unanticipated shocks that are announced in period $t = 0$, at which point all agents share the same unconditional distribution for $\omega \sim N(\mu_0, \tau_0^{-1})$. Starting in period $t = 1$, informed agents receive signals $s_t = \omega + \epsilon_t$ where $\epsilon_t \sim (N, \tau_s^{-1})$ each period. Uninformed agents do not observe s_t, but can learn information from past prices.

For tractability, we assume that no agent uses the history of D_t to learn information about g_t. This assumptions allows us to not have to deal with an additional signal that agents receive about g_t, and which they would be combining with the information they either receive or learn from past prices.[5] One way to rationalize this is to think of σ_u^2 as being extremely large, so that ΔD_t provides too noisy a signal of g_{t+1}.

D.3.1 Normal Times

Informed Agents' Beliefs. In normal times informed agents' beliefs are given by:

$$\mathbb{E}_{I,t}[D_{t+1}] = D_t + g_{t+1} \tag{D.96}$$

[5]We have run simulations where agents also use the history of past dividends to learn about the growth rate, and found similar results.

$$\mathbb{V}_I[D_{t+1}] = \sigma_\xi^2 \tag{D.97}$$

PET Agents' Beliefs and Equilibrium Prices. We assume that PET agents learn information from prices under the mistaken belief that all other agents do not infer information from prices. They thus believe that other uninformed agents have the following beliefs:

$$\tilde{\mathbb{E}}_{U,t}[D_{t+1}] = D_t + \bar{g} \tag{D.98}$$

$$\tilde{V}_{U,t}[D_{t+1}] = \frac{\sigma_u^2}{1-\rho^2} + \sigma_\xi^2 \tag{D.99}$$

This implies that they believe that the equilibrium price is formed according to:

$$\tilde{P}_t = D_t + \tilde{a} g_{t+1} + \tilde{b}\bar{g} - \tilde{c} \tag{D.100}$$

where $\tilde{a} = \left(\frac{\phi \tilde{V}_{U,t}}{\phi \tilde{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$, $\tilde{b} = \left(\frac{(1-\phi)\mathbb{V}_{I,t}}{\tilde{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$, $\tilde{c} = \left(\frac{AZ\mathbb{V}_{I,t}\tilde{V}_{U,t}}{\phi \tilde{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$.

Uninformed agents then extract $\tilde{g}_t$ from prices as follows:

$$\tilde{g}_t = \frac{P_{t-1} - D_{t-1} - \tilde{b}\bar{g} + \tilde{c}}{\tilde{a}} \tag{D.101}$$

This leads to PET agents holding the following posterior beliefs:

$$\mathbb{E}_{U,t}[D_{t+1}] = D_t + (1-\rho)\bar{g} + \rho\tilde{g}_t \tag{D.102}$$

$$\mathbb{V}_{U,t}[D_{t+1}] = \sigma_u^2 + \sigma_\xi^2 \tag{D.103}$$

The PET equilibrium price is then given by:

$$P_t = D_t + a g_{t+1} + b\left((1-\rho)\bar{g} + \rho\tilde{g}_t\right) - c \tag{D.104}$$

where $a = \left(\frac{\phi\mathbb{V}_{U,t}}{\phi\mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$, $b = \left(\frac{(1-\phi)\mathbb{V}_{I,t}}{\phi\mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$, $c = \left(\frac{AZ\mathbb{V}_{I,t}\mathbb{V}_{U,t}}{\phi\mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)$. Rearranging this expression, and using the results above, we can rewrite the price dividend ratio as:

$$(P_t - D_t) - \overline{P - D} = a \frac{u_{t+1}}{(1 - \rho\mathbb{L})\left(1 - \frac{\rho b}{\tilde{a}}\mathbb{L}\right)} \tag{D.105}$$

where $\overline{P - D} = (\bar{g} - c) - \frac{b\rho}{\tilde{a}}(\bar{g} - \tilde{c})$. This expression in an AR(2). For the price dividend ratio to be stationary in normal times, we need both roots of the autoregressive coefficients to lie outside the unit circle: $\rho < 1$ and $\frac{\rho b}{\tilde{a}} < 1$.

REE Agents' Beliefs and Equilibrium Prices. Finally, rational uninformed agents also learn from past prices, but are able to extract the right information from them.

$$\mathbb{E}^{REE}_{U,t}[D_{t+1}] = D_t + (1 - \rho)\bar{g} + \rho g_t \tag{D.106}$$

$$\mathbb{V}^{REE}_{U,t}[D_{t+1}] = \mathbb{V}_{U,t}[D_{t+1}] \tag{D.107}$$

The REE equilibrium is then given by:

$$P^{REE}_t = D_t + a g_{t+1} + b((1 - \rho)\bar{g} + \rho g_t) - c \tag{D.108}$$

where a, b and c are the same coefficients as in PET, as agents have the same conditional variance in PET and REE.

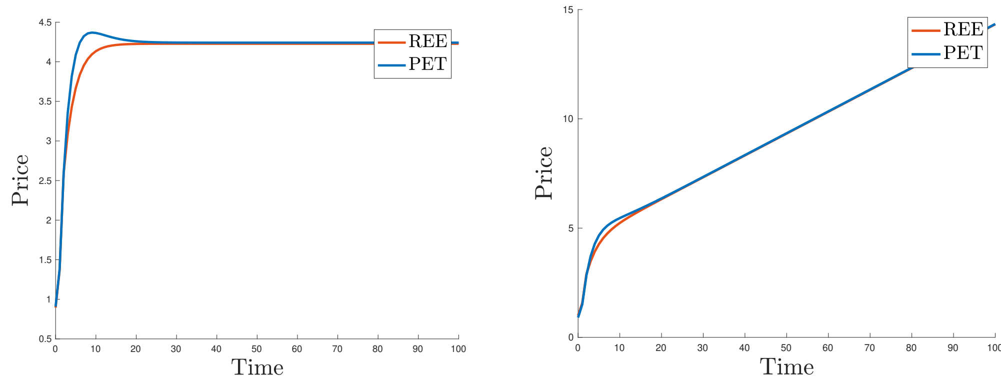
Rearranging and using the results above, we see that in normal times REE prices evolve according to an ARMA(1,1), which is stationary as long as $\rho < 1$:

$$(P^{REE}_t - D_t) - \overline{P - D}^{REE} = a \frac{\left(1 + \frac{b\rho}{a}\mathbb{L}\right)}{(1 - \rho\mathbb{L})} u_{t+1} \tag{D.109}$$

where $\overline{P - D}^{REE} = \bar{g} - c$.

Simulation. Figure 7 simulates the path of equilibrium prices when $\bar{g} = 0$ and $\bar{g} > 0$. Regardless of $\bar{g}$, PET leads to mild momentum and reversals in normal times.

Figure 7: Path of equilibrium prices in normal times with an unobservable growth rate of dividend. In the left panel $\bar{g} = 0$, while in the right panel $\bar{g} > 0$.



D.3.2 Displacement

Shock. Starting from the normal times steady state, suppose a displacement shifts the unconditional mean of the growth rate of dividends from $\bar{g}$ to $\bar{g} + \frac{\omega}{1-\rho}$.

$$D_{t+1} = D_t + g_{t+1} + \xi_{t+1} \tag{D.110}$$

$$g_{t+1} = (1-\rho)\bar{g} + \rho g_t + \omega + u_{t+1} \tag{D.111}$$

In period $t = 0$, all agents learn about the existence of this shock, and have the same unconditional prior over it $\omega \sim N(\mu_0, \tau_0^{-1})$. Starting in period $t = 1$, informed agents receive signals $s_t = \omega + \epsilon_t$ each period, where $\epsilon_t \sim N(0, \tau_s^{-1})$. Uninformed agent do not observe this signal, and instead continue to learn information from past prices.

Period $t > 1$. To solve the model for period $t > 1$ it is convenient to rewrite the process for dividends conditional on the information set in period $t = 0$:

$$D_{t+1} = D_t + (1-\rho^{t+1})\left(\bar{g} + \frac{\omega}{1-\rho}\right) + \rho^{t+1} g_0 + U_{t+1} + \xi_{t+1} \tag{D.112}$$

where $U_{t+1} = \sum_{j=0}^{t} \rho^j u_{t+1-j} = \rho^t u_1 + \sum_{j=1}^{t-1} \rho^j u_{t+1-j} + u_{t+1} = \rho U_t + u_{t+1}$.

Informed Agents. In period $t > 2$, informed agents' beliefs are given by:

$$\mathbb{E}_{I,t}[D_{t+1}] = D_t + \underbrace{(1-\rho^{t+1})\left(\bar{g} + \frac{\frac{t\tau_s}{t\tau_s+\tau_0}S_t + \frac{\tau_0}{t\tau_s+\tau_0}\mu_0}{1-\rho}\right) + \rho^{t+1}g_0 + U_{t+1}}_{g_{t+1|t}} \tag{D.113}$$

$$\mathbb{V}_{I,t}[D_{t+1}] = \left(\frac{1-\rho^{t+1}}{1-\rho}\right)^2 (t\tau_s+\tau_0)^{-1} + \sigma_\xi^2 \tag{D.114}$$

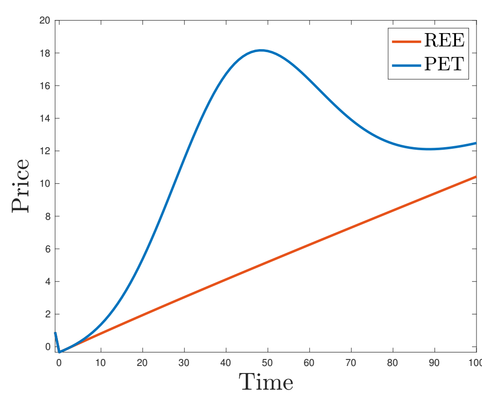
PET Agents' Beliefs and Equilibrium. PET agents' beliefs are given by:

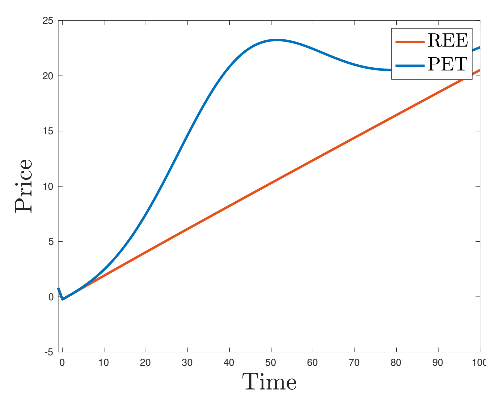
$$\mathbb{E}_{U,t}[D_{t+1}] = D_t + (1-\rho^{t+1})\left(\bar{g} + \frac{\mathbb{E}_{U,t}[\omega|\tilde{g}_{t|t-1}]}{1-\rho}\right) + \rho^{t+1}\tilde{g}_0 + \rho^t\tilde{u}_1 + \rho\mathbb{E}_{U,t}[U_t|\tilde{g}_{t|t-1}] \tag{D.115}$$

$$\begin{aligned}\mathbb{V}_{U,t}[D_{t+1}] = &\left(\frac{1-\rho^{t+1}}{1-\rho}\right)^2 \mathbb{V}_{U,t}[\omega|\tilde{g}_{t|t-1}] + \rho^2\mathbb{V}_{U,t}[U_t|\tilde{g}_{t+1|t}] \\ &+ 2\left(\frac{1-\rho^{t+1}}{1-\rho}\right)\rho\mathrm{Cov}_{U,t}(\omega, U_t|\tilde{g}_{t|t-1}) + \sigma_u^2 + \sigma_\xi^2\end{aligned} \tag{D.116}$$

Simulation. Given these beliefs, Figure 8 simulates the path of equilibrium prices when $\bar{g} = 0$ and $\bar{g} > 0$. Parameters are the same as in normal times. Even with this setup, PET delivers bubbles and crashes, and results are robust to the initial level of $\bar{g}$.

Figure 8: Path of equilibrium prices following a displacement which shocks the growth rate of dividends from $\bar{g}$ to $\bar{g} + \frac{\omega}{1-\rho}$. In the left panel $\bar{g} = 0$, while in the right panel $\bar{g} > 0$.





E Market Orders

In this section we consider the case where uninformed traders submit market orders, so that they do not condition on current prices in computing their demand for the risky asset. When this is the case, uninformed traders effectively end up changing the net supply of the risky asset available to the informed traders. Partial equilibrium thinkers then think that other uninformed traders hold a constant amount in the risky asset and that the net supply available to informed traders is fixed, when in reality it is time-varying as uninformed traders update their demand based on information they learn from past prices.

While the exact functional form of the results changes, the key intuitions and results from the baseline model still go through. Specifically, partial equilibrium thinkers still generate a bias that is decreasing in the perceived informational edge of informed traders, and it still leads to constant price extrapolation in normal times, and time-varying extrapolation following a displacement.

E.1 Setup

We maintain the same assumptions about the setup and information structure as in the baseline model. Specifically, in each period t, informed traders receive signals about the terminal dividend, and uninformed traders can learn information from past prices.

The only difference to our baseline model is that we now assume that uninformed traders do not condition on current prices, and instead submit market orders, and submit the following demand for the risky asset (Kyle 1985, Campbell and Kyle 1993, Campbell 2017):

$$X_{U,t} = \frac{\mathbb{E}_{U,t}[D_T]}{\mathcal{A}\mathbb{V}_{U,t}[D_T]} \tag{E.1}$$

To solve the model we then take similar steps as in the main text. First, we compute the true price function, conditional on traders' posterior beliefs. Second, we compute the price function which uninformed traders think is generating the price changes they observe, and which they use to infer information from prices. Third, we combine these two mappings and consider the properties of equilibrium outcomes.

We first solve the model in normal times, and then add displacement shocks.

E.2 Normal Times

In this section we show that, even when uninformed traders submit market orders, in normal times: i) partial equilibrium thinkers still extrapolate recent price changes they observe, ii) the bias is still decreasing in informed traders' informational edge, and iii) stationarity still requires the aggregate confidence of informed traders to be greater than the aggregate confidence of uninformed traders.

Step 1: True Market Clearing Price Function. The market clearing condition which equates the aggregate demand for the risky asset to the fixed supply is given by:

$$\phi\left(\frac{\mathbb{E}_{I,t}[D_T]-P_t}{\mathcal{A}\mathbb{V}_I}\right)+(1-\phi)\left(\frac{\mathbb{E}_{U,t}[D_T]}{\mathcal{A}\mathbb{V}_U}\right)=Z \tag{E.2}$$

where $\mathbb{V}_I = \mathbb{V}_{I,t}[D_T]$ and $\mathbb{V}_U = \mathbb{V}_{U,t}[D_T]$ are constant and equal to the normal time variances we had in the baseline model in (5) and (7), respectively. Solving for P_t, and using the definition of the aggregate informational edge of informed traders relative to uninformed traders, $\zeta = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U}{\mathbb{V}_I}\right)$, we find that the true price function, conditional on agents' posterior beliefs, is given by the following expression:

$$P_t = \mathbb{E}_{I,t}[D_T] + \frac{1}{\zeta_t}\mathbb{E}_{U,t}[D_T] - \frac{\mathcal{A}\mathbb{V}_I}{\phi}Z \tag{E.3}$$

Taking first differences, and using the fact that $\Delta\mathbb{E}_{I,t}[D_T] = u_t$ and $\Delta\mathbb{E}_{U,t}[D_T] = \tilde{u}_{t-1}$, we find that price changes reflect changes in beliefs of both informed and uninformed traders, just as in the baseline model:

$$\Delta P_t = u_t + \frac{1}{\zeta}\tilde{u}_{t-1} \tag{E.4}$$

Step 2: Partial Equilibrium Thinking Mapping. Partial equilibrium thinkers think that other uninformed traders do not learn information from prices, and trade on their unconditional prior beliefs. Since uninformed traders learn information from past prices, we consider the market clearing condition for period $t-1$, as this provides us with an expression for P_{t-1}, the price they are learning from in period t:

$$\phi\left(\frac{\tilde{\mathbb{E}}_{I,t-1}[D_T]-P_{t-1}}{\mathcal{A}\tilde{\mathbb{V}}_I}\right)+(1-\phi)\left(\frac{\bar{D}}{\mathcal{A}\tilde{\mathbb{V}}_U}\right)=Z \tag{E.5}$$

Solving for P_{t-1}, and using the definition of the perceived informational edge as in our main setup, $\tilde{\zeta}\equiv\left(\frac{\phi}{1-\phi}\right)\left(\frac{\tilde{\mathbb{V}}_U}{\tilde{\mathbb{V}}_I}\right)$, we obtain the following perceived price function:

$$P_t=\tilde{\mathbb{E}}_{I,t-1}[D_T]+\frac{1}{\tilde{\zeta}}\bar{D}-\frac{\mathcal{A}\mathbb{V}_I}{\phi}Z \tag{E.6}$$

Taking first differences, and using the fact that $\Delta\tilde{\mathbb{E}}_{I,t-1}=\tilde{u}_{t-1}$, we see that partial equilibrium thinker still attribute every price change to new information alone, as in the baseline model:

$$\Delta P_t=\tilde{u}_{t-1} \tag{E.7}$$

Partial equilibrium thinkers then trivially invert this mapping to extract the following signal from past price changes they observe:

$$\tilde{u}_{t-1}=\Delta P_{t-1} \tag{E.8}$$

so that they still extrapolate price changes they observe, and the fact that they extrapolate one-to-one simply reflects that informed traders' beliefs are now incorporated into prices one-to-one.[6]

[6] We can compare this to the rational benchmark where uninformed traders understand what generates the price changes they observe, and use the following mapping in their inference:

$$\tilde{u}^{REE}_{t-1}[D_T]=\Delta P_{t-1}-\frac{1}{\zeta}\tilde{u}_{t-2} \tag{E.9}$$

Comparing (E.8) to (E.9), we notice that, just as in the baseline model, the bias inherent in partial equilibrium thinking doesn't come directly from the weight that uninformed traders put on past price changes (in this case 1), but rather it comes from the part of the price variation they neglect. Specifically, rational uninformed traders do condition on past price changes, but the mapping they use also has a correction term to account for the fact that part of the price change they observe comes from the lagged response of all other uninformed traders who are also learning information from prices with a lag, as

Step 3: Properties of Equilibrium Outcomes. Combining the results in (E.4) and (E.8), we find that changes in prices and in beliefs evolve as follows:

$$\Delta P_t = u_t + \frac{1}{\zeta}\Delta P_{t-1} \tag{E.10}$$

$$\tilde{u}_{t-1} = u_{t-1} + \frac{1}{\zeta}\tilde{u}_{t-2} \tag{E.11}$$

which closely mirrors the expressions in (24) and (25) in the baseline model. Specifically, (E.11) shows that the bias in the signal uninformed traders extract from past prices $\tilde{u}_{t-1} - u_{t-1}$ is still decreasing in informed traders' informational edge, and the AR(1) coefficient in (E.10) and (E.11) shows that in normal times stationarity still requires that the $\zeta < 1$, or that the aggregate confidence of informed traders be greater than the aggregate confidence of uninformed traders, as in the baseline model.

E.2.1 Displacements

In this section, we introduce displacement shocks as in (33), and show that, even when uninformed traders can only submit market orders, i) partial equilibrium thinking still leads to time-varying price extrapolation, and that ii) local stationarity depends on the true informational edge.

Step 1: True Market Clearing Price Function. The market clearing condition which equates the aggregate demand for the risky asset to the fixed supply is now given by:

$$\phi\left(\frac{\mathbb{E}_{I,t}[D_T] - P_t}{\mathcal{A}\mathbb{V}_{I,t}[D_T]}\right) + (1-\phi)\left(\frac{\mathbb{E}_{U,t}[D_T]}{\mathcal{A}\mathbb{V}_{U,t}[D_T]}\right) = Z \tag{E.12}$$

Solving for P_t, and using the the definition of the aggregate informational edge of informed traders relative to uninformed traders: $\zeta_t = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_{U,t}[D_T]}{\mathbb{V}_{I,t}[D_T]}\right)$, we obtain the true price function, conditional on agents' posterior beliefs:

$$P_t = \mathbb{E}_{I,t}[D_T] + \frac{1}{\zeta_t}\mathbb{E}_{U,t}[D_T] - \frac{\mathcal{A}\mathbb{V}_{I,t}[D_T]}{\phi}Z \tag{E.13}$$

shown in the second term in (E.9), which is instead missing in the PET mapping in (E.8).

Step 2: Partial Equilibrium Thinking Mapping. Partial equilibrium thinkers think that other uninformed traders do not learn information from prices, and trade on their unconditional prior beliefs. Therefore, they think that P_{t-1} (the price they are learning from in period t) is determined from the following market clearing condition:

$$\phi\left(\frac{\tilde{\mathbb{E}}_{I,t-1}[D_T]-P_{t-1}}{\mathcal{A}\tilde{\mathbb{V}}_{I,t-1}[D_T]}\right)+(1-\phi)\left(\frac{\bar{D}+\mu_0}{\mathcal{A}\tilde{\mathbb{V}}_{U,t-1}[D_T]}\right)=Z \tag{E.14}$$

Solving for P_{t-1}, and using the definition of the perceived informational edge as in our main setup, $\tilde{\zeta}_t \equiv \left(\frac{\phi}{1-\phi}\right)\left(\frac{\tilde{\mathbb{V}}_{U,t}[D_T]}{\tilde{\mathbb{V}}_{I,t}[D_T]}\right)$, we obtain the following perceived price function:

$$P_{t-1}=\tilde{\mathbb{E}}_{I,t-1}[D_T]+\frac{1}{\tilde{\zeta}_{t-1}}\left(\bar{D}+\mu_0\right)-\frac{\mathcal{A}Z}{\phi}\mathbb{V}_{I,t-1} \tag{E.15}$$

where we also define $\mathbb{V}_{i,t-1}\equiv\mathbb{V}_{i,t-1}[D_T]$ for $i\in\{I,U\}$, for ease of notation. Taking first differences, and rearranging, we find that partial equilibrium thinkers still extrapolate unexpected price changes:

$$\Delta\mathbb{E}_{U,t}[D_T]=\Delta P_{t-1}+\left(\frac{\Delta\tilde{\zeta}_{t-1}}{\tilde{\zeta}_{t-1}\tilde{\zeta}_{t-2}}\right)\left(\bar{D}+\mu_0\right)+\frac{\mathcal{A}Z}{\phi}\Delta\mathbb{V}_{I,t-1} \tag{E.16}$$

Notice that while the degree of price extrapolation is still 1, this is still not the same as constant price extrapolation, since the second and third terms in the above expressions are still time-varying (which wouldn't be the case with constant price extrapolation).[7]

[7] We can once again compare this to the rational benchmark, where uninformed traders take into account that other uninformed traders are also learning information from past prices. In this case, uninformed traders' changes in beliefs would evolve as follows:

$$\Delta\mathbb{E}_{U,t}[D_T]=\Delta P_{t-1}+\left(\frac{\Delta\zeta_{t-1}}{\zeta_{t-1}\zeta_{t-2}}\right)\mathbb{E}_{U,t-1}[D_T]-\frac{1}{\zeta_{t-2}}\Delta\mathbb{E}_{U,t-1}[D_T]+\frac{\mathcal{A}Z}{\phi}\Delta\mathbb{V}_{I,t-1} \tag{E.17}$$

and we can re-write this expression in a way that highlights the source of price variation that partial equilibrium thinkers neglect:

$$\Delta\mathbb{E}_{U,t}[D_T]=\Delta P_{t-1}+\left(\frac{\Delta\zeta_{t-1}}{\zeta_{t-1}\zeta_{t-2}}\right)\bar{D}+\underbrace{\left(\frac{\Delta\zeta_{t-1}}{\zeta_{t-1}\zeta_{t-2}}\right)\left(\mathbb{E}_{U,t-1}[D_T]-\left(\bar{D}+\mu_0\right)\right)-\frac{1}{\zeta_{t-2}}\Delta\mathbb{E}_{U,t-1}}_{\text{source of price variation PET traders neglect}}+\frac{\mathcal{A}Z}{\phi}\Delta\mathbb{V}_{I,t-1} \tag{E.18}$$

As in the baseline framework, this bias is time-varying following a displacement.

Step 3: Properties of Equilibrium Outcomes. Whether the price function is in a stationary or non-stationary region now purely depends on the true informational edge:

$$\Delta P_t = \Delta \mathbb{E}_{I,t}[D_T] + \frac{1}{\zeta_t}\Delta \mathbb{E}_{U,t}[D_T] + \Delta\left(\frac{1}{\zeta_t}\right)\mathbb{E}_{U,t-1}[D_T] - \frac{\Delta \mathbb{V}_{I,t}}{\phi}\mathcal{A}Z \tag{E.19}$$

which we can re-write as:

$$\begin{aligned}\Delta P_t = \Delta \mathbb{E}_{I,t}[D_T] &+ \frac{1}{\zeta_t}\Delta P_{t-1} \\ &+ \frac{1}{\zeta_t}\left(\frac{\Delta\tilde{\zeta}_{t-1}}{\tilde{\zeta}_{t-1}\tilde{\zeta}_{t-2}}\right)\left(\bar{D}+\mu_0\right) + \Delta\left(\frac{1}{\zeta_t}\right)\mathbb{E}_{U,t-1}[D_T] \\ &\qquad\qquad - \frac{\mathcal{A}Z}{\phi}\Delta\mathbb{V}_{I,t} + \frac{1}{\zeta_t}\frac{\mathcal{A}Z}{\phi}\Delta\mathbb{V}_{I,t-1}\end{aligned} \tag{E.20}$$

However, deviations from rationality still depend both on the true and the perceived informational edges. An intuitive way to see this is to express the difference between uninformed traders' beliefs at t and informed traders' beliefs at $t-1$. In the rational benchmark, that difference is simply 0. Instead, when traders think in partial equilibrium, this difference is given by:

$$\mathbb{E}_{U,t}[D_T] - \mathbb{E}_{I,t-1}[D_T] = \frac{\mathbb{E}_{U,t-1}[D_T]}{\zeta_{t-1}} - \frac{\bar{D}+\mu_0}{\tilde{\zeta}_{t-1}} \tag{E.21}$$

which depends both on the true and perceived informational edges, as well as past PET beliefs.

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